AN EXPLORATORY INPUT-OUTPUT MODEL FOR THE RURAL SECTOR IN VIETNAM

Do Vo Quang

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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by

Do Vo Quang

Thesis Advisor: N. Keith Womer

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Do Vo Quang
2Lt, Army of the Republic of Vietnam
B.S., University of Saigon, 1968

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ABSTRACT

An exploratory approach to the problem of modeling the rural sector in Vietnam as an open static input-output model is attempted. A tentative classification of the agricultural economy into primary and secondary sectors is undertaken. Methods for data collection and uses are suggested. Anticipating the case where data for more than one time period are available a statistical estimation procedure for parameters is developed; a time series analysis of technical coefficients is also presented. The uses of the model as a predicting device is discussed and a linear programming approach to the problems of resources allocation is proposed.

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I. INTRODUCTION

Agriculture plays a primordial role in the economy of the Republic of Vietnam. It employs 70 percent of the total work force; it accounts for more than 30 percent of the GNP. Export of agricultural products in the form of rubber, spices, fish products, tea, coffee, and wood products constitutes 90 percent of annual national foreign income. The windingdown of the war and the increase in security that will result in the countryside will permit a rapid expansion of the cultivated area. Thus a rapid increase in agricultural output is expected to last through 1978.

In an effort to equalize resource allocations and promote a smooth growth for the rural sector, a five-year plan was adopted for the period 1971-1975. The plan defined two major objectives for the agricultural sector:

1. Increase in food production so as to meet national consumption.

2. Increase in production of potentially exportable crops for export purposes in order to earn foreign currency needed to finance industrial development.

The plan started from scratch and derived its policies from specific project plans prepared for each of the various sectors of the agriculture. The projections were based on a linear regression trend line. Although those predictions to date have proved themselves to be fairly accurate, the plan has been criticized as being over-optimistic and neglecting the side effects of different economic measures. It is notorious that planners for economic policies have the tendency to concentrate their attention on the principal effects of economic measures and neglect the less conspicuous indirect effects which, in some cases, can hinder growth in other areas of the economy. Subsidies for export encourage the production

of exportable crops such as rice, tea, coffee, and rubber but at the same time reduce land allocation for other crops, soybeans for example, which in turn affects the production of pigs and poultry.

Clearly, there is a need for an economic model which is capable of accounting for the interdependency among various sectors of agriculture. Input-output models are such models. The use of input-output models as planning tools have received much attention during recent years, particularly in developing countries. Economic successes in India,[1] Korea,[2] and other countries are attributed to the economic measures derived from input-output analysis. The input-output model permits the disentanglement and the accurate measurement of the indirect effects and thus gives a better insight of the structure of the economy. These properties should logically lead to better plannings.

This paper represents an effort to model the Vietnamese rural sector as an input-output model and is divided into six sections. The first two sections present the problem and review the theoritical static input-output models.

The problems that arise in input-output modeling are examined in section III. These problems are of various nature and are generally classified under two main categories: the aggregation problems are encountered early during the conceptual partitioning of the economy, and the problems of coefficients estimation that arise during the construction of the technology matrix.

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[1] Adelman, Irma, Practical Approaches To Development Planning: Korea's Second Five-Year Plan, p. 3-10, The Johns Hopkins Press, Baltimore, 1969.

[2] Bharadwaj, R., Economic Analysis in Input-Output Framework, Input-Output Research Association, India, 1967.

Section IV discusses the statistical estimation procedures that can be used in cases where data for more than one time period are available.

In section V, the computational procedures for the derivation of input-output projections are presented. A linear programming approach to the problem of optimum resources allocation is also proposed.

Section VI reviews the assumptions of the model and discusses the cases where those assumptions are violated.

II. THE STATICAL LEONTIEF MODEL

The Statical Input-Output Model is essentially a system of simultaneous linear production functions describing an economy considered as a unit.

The theoretical ground is developed upon three basic assumptions:

1. Constant return to scale. The inputs to each industry are a unique function of the level of output of that sector.

2. No joint products, no process produces more than one product.

3. No substitution among inputs is possible in the production of any goods or services.

A. THE LEONTIEF CLOSED MODEL[3]

Consider an exhaustive partition of an economy into n producing industries.

Let X_i be the total gross output of the ith industry.

$$i = 1, 2, \ldots, n$$

x_{ij} be the input of the ith product used by the jth industry to produce X_j.

$$i = 1, 2, \ldots, n$$

$$j = 1, 2, \ldots, n$$

D_i be the output of industry i available for outside consumption or Final Demand.

Then the balance equation for commodity i can be written as:

$$(2.1) \quad X_i = \sum_{j=1}^{n} x_{ij} + D_i \qquad i = 1, 2, \ldots, n$$

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[3] Dorfman, R.P. Samuelson, and R. Solow, Linear Programming and Economic Analysis, pp. 204-229, McGraw Hill Book Company, New York, 1959. ALSO Lancaster, Kelvin, Mathematical Economics, The Macmillan Company, Collier-Macmillan Limited, London, 1968.

Equation (2.1) is a system of n equations which can be conveniently represented in the form of a table called The Transactions Table (Table 2.1).

Industries Producing	Industries Purchasing 1	2	. . .	n	Final Demand	Gross Output
1	x_{11}	x_{12}	. . .	x_{1n}	D_1	X_1
2	x_{21}	x_{22}	. . .	x_{2n}	D_2	X_2
.	.	.	. . .	.	.	.
.	.	.	. . .	.	.	.
.	.	.	. . .	.	.	.
n	x_{n1}	x_{n2}	. . .	x_{nn}	D_n	X_n

Table 2.1. Transactions Table for the Closed Model

Given the assumptions enumerated above, the input x_{ij} required to produce X_j is a unique linear function of X_j. Then

(2.2) $$x_{ij} = a_{ij}X_j \quad \text{or} \quad a_{ij} = \frac{x_{ij}}{X_j}$$

where a_{ij} is the quantity of input x_{ij} required to produce one unit of output X_j.

Using relation (2.2) and replacing x_{ij} in system of equations (2.1) gives

(2.3) $$X_i = \sum_{j=1}^{n} a_{ij}X_j + D_i \ ; \ i = 1, 2, \ldots, n$$

Let $A = (a_{ij})$ be the technology matrix

$X = (X_1, X_2, \ldots, X_n)$

and $D = (D_1, D_2, \ldots, D_n)$

Then (2.3) can be rewritten as:

(2.4) $X = AX + D$

or $(I-A)X = D$

And if $(I-A)^{-1}$ denotes the inverse of matrix $(I-A)$ then

(2.5) $X = (I-A)^{-1} D$

Thus given the technology matrix A and the vector of gross output X (2.4) gives the vector of final demand.

Similarly, given the final demand vector D equation (2.5) gives the vector of gross output.

It may be noted that the inverse of matrix (I-A) always exists if the Hawkins-Simon conditions are satisfied.

B. THE HAWKINS-SIMON CONDITION[4]

If X is the given output vector, then the vector of all inputs to the system necessary to produce X is AX. Since the only inputs to the system are also the outputs, it cannot produce and operate unless:

$$X \geq AX \quad \text{or} \quad (I-A)\, X \geq 0$$

In words, this says that the production of any one unit of good must not require directly or indirectly more than one unit of itself.

The Hawkins-Simon condition expressed in matrix form is as follows: an input-output system is viable if the ith principal minor of the matrix (A-I) has for sign $\mathrm{sgn}(-1)^i$ for $i = 1, 2, \ldots, n$.

Consider a 2 X 2 case.

Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{be the technology matrix}$$

The Hawkins-Simon conditions are:

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[4] Lancaster, K., Mathematical Economics, pp. 80-81, ALSC Appendix R.7, pp. 305-317.

$$1 - a_{11} > 0 \qquad 1 - a_{22} > 0$$

$$\text{Det} \begin{vmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{vmatrix} > 0$$

If the Hawkins-Simon conditions are satisfied, then Det(I-A) is nonzero, the inverse $(I-A)^{-1}$ exists.[5]

C. THE LEONTIEF OPEN MODEL

The closed Leontief Model can be "opened" by introducing a primary input sector.

A primary input is a commodity which is not an output of any production process of the system; it can be identified with labor, imports, or any scarce factor found to be important to the analysis of the model.

The system is now divided into two sectors: the primary sector which does not contain any producing industries and the secondary sector which contains all the producing industries of the system.

Let Y_k be the total amount of the kth primary input available

and y_{kj} be the amount of primary input k used by the jth industry.

Then:

$$\sum_{j=1}^{n} y_{k,j} \leq Y_k \qquad k = 1, 2, \ldots, m$$

Define $b_{kj} = \dfrac{y_{kj}}{X_j}$ as the amount of primary input k required per unit output j.

Then $$\sum_{j=1}^{n} b_{kj} X_j \leq Y_k; \; k = 1, 2, \ldots, m.$$

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[5]An approximation of the inverse $(I-A)^{-1}$ can be computed as follows: $(I-A)^{-1} = I + A + A^2 + A^3 + \ldots$ A numerical example is presented in Appendix B.

Thus if the total quantity of primary input k is limited to a level Y_k^o, then only a certain proportion of output X can be produced and therefore only a proportion of the final demand can be attained.

The standard transactions table for the open model is given in Table (2.2).

D. INPUT-OUTPUT TABLES IN VALUE TERMS

In the theoretical input-output table, the numbers being reported represent physical units: x_{ij} is the amount of commodity j used to produce X_j; consequently, $a_{ij} = \frac{x_{ij}}{X_j}$ is the ratio of two distinct physical commodities. This creates an awkward situation where a large amount of numbers, the a_{ij}'s, must be carried along with two units each. Moreover, experience has shown that data in physical units are difficult to obtain and usually transactions are recorded in monetary value.[6] For these reasons, actual input-output tables are constructed in terms of money values rather than in terms of physical units.

Let P_i be the price of a unit of commodity X_i

V_i be the money value of the total of the ith industry

v_{ij} be the value of sales of industry i to industry j.

Then

$$v_{ij} = P_i \, x_{ij} \qquad \text{and} \qquad V_j = P_j \, X_j$$

Constructing the coefficients a'_{ij} as:

$$a'_{ij} = \frac{v_{ij}}{V_j} = \frac{P_i x_{ij}}{P_j X_j} = \frac{P_i}{P_j} a_{ij}$$

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[6]Gosfield, Amor, Input-Output Analysis of the Puerto Rican Economy, Input-Output Analysis: An Appraisal, pp. 321-367.

If prices are assumed constants in the time period covered by the table, then a'_{ij} reflects the technology of industry j.

a'_{ij} = money value of input i to the jth industry to produce one unit monetary value of output j

Note that if equilibrium condition and zero profit are assumed, then the total value of inputs to any industry should add up to the value of output of that industry (viability condition) or:

$$(2.6) \quad \sum_{i=1}^{n} v_{ij} = V_j$$

and $$\sum_{i=1}^{n} a'_{ij} = 1.$$

		Purchasing Industries 1	2	. . .	n	Export	Con-sump-tion	Total Demand	Gross Output
Primary Sector Producing Industries	1	x_{11}	x_{12}	. . .	x_{1n}	d_{11}	d_{12}	D_1	X_1
	2	x_{21}	x_{22}	. . .	x_{2n}	d_{21}	d_{22}	D_2	X_2
	.	.			.	.	.	.	.
	.	.			.	.	.	.	.
	.	.			.	.	.	.	.
	n	x_{n1}	x_{n2}	. . .	x_{nn}	d_{n1}	d_{n2}	D_n	X_n
Primary Sector	1	y_{11}	y_{12}	. . .	y_{1n}	0	0	0	Y_1
	.	.			.				.
	.	.			.				.
	.	.			.				.
	m	y_{m1}	y_{m2}	. . .	y_{mn}	0	0	0	Y_m
Total Output		X_1	X_2	. . .	X_n				

Table 2.2. Transaction Table for the Open Model

III. INPUT-OUTPUT MODEL FOR THE RURAL SECTOR

An important and perhaps most sensitive step in input-output modeling is the construction of the Transactions Table. It consists of enumerating the different industries to be allocated to the rows and columns and defining the kind of data needed to fill in the slots of the table. In the following, these problems will be discussed and a tentative industrial classification will be attempted.

A. SECTORS OF THE AGRICULTURE.

The partitioning of an economy into industries is subject to a certain number of theoretical and practical problems. The aggregation problems derive directly from the assumptions of the model. To minimize possible deviations from those assumptions implies a partitioning of the economy into an infinite number of industries, where each producing unit is being considered as an industry. Clearly, this is infeasible. Invariably, computational capacity constraints impose some level of aggregation. And, when n producing units are aggregated into one industry or sector, it can happen that one industry produces more than one product, in which case assumption (2) on joint products is violated, or it can happen that the sector uses more than one process, then assumption (1) on constant production coefficients is violated. In practice, model builders try to construct industrial sectors in such a way to obtain sectors as homogeneous as possible. A number of techniques for aggregation were developed by Mathilda Holzman.[7] More recently, an interesting scheme for industrial

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[7] Leontief, Wassily, Studies in the Structure of the American Economy, pp. 326-359.

aggregation was developed by V. Kossov.[8] In the present case, it is conceivable that those problems are rarely critical, the structure of agriculture being relatively simple.

In 1960, Michio Hatanaka suggested three bases for industrial classification:

1. Commodity basis
2. Activity basis
3. Establishment basis[9]

Again, because of the structural simplicity of agriculture and adding to the fact that most agricultural data are being compiled based on outputs of commodities by type, a classification based on commodity basis seems to be a natural choice.

The partition still depends on considerations with respect to the underlying objectives and the size of the model. An input-output model may embody a classification system that ranges from fine detail to a high degree of aggregation. At this point, it may be noted that every aggregation sacrifices a part of the details relative to the structure of the system, and as it has been pointed out every condensation of industries increase the deviation from the basic assumptions of the model.

On the other hand, a classification of n sectors requires the estimation of n x (n+1) data points: corresponding to each sector, n values for flows and the gross output must be determined. From there, any further disaggregation increases the number of parameters to be estimated by 2 x (n+1). Accordingly, the labor requirements to the following stages of the analysis will increase considerably. There will be then a constant

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[8] Kossov, V., Contributions to Input-Output Analysis, edited by A.P. Carter and A. Brody, chap. 7, pp. 241-248.

[9] Michio Hatanaka, The Workability of Input-Output Analysis, pp. 15-20.

conflict between the desire to retain in the analysis all possible relevant details by distinguishing a maximum number of sectors and the need to conform to the limited resources allocated to model building.

Considering that in any developing country resource limitations are by far the most important constraint to any research, the number of sectors to be distinguished must necessarily be small. A reasonable model for the Rural Sector would encompass a classification of no more than thirty sectors.

There is also a strong incentive to make use of the data routinely compiled by existing governmental agencies. This in some cases means a partition tailored to the existing data.

The above considerations by no means exhaust all the problems relative to model building. The input-output analysis puts a primary importance on inter-industry flows; therefore, an extensive knowledge of agricultural processes is required on the part of the investigator. The model builder should be able to detect bottlenecks that would develop if an intermediate sector whose output may be modest but provides necessary inputs to other sectors is neglected.

In the analysis which follows, a partition is suggested taking as objectives those that have been delineated by the five-year plan. Namely:

1. Increase in production to satisfy consumption.
2. Increase in production of exportable crops.

The criteria used in the partition will then be:

a. Size of output

It is implicitly assumed that the size of output of a commodity reflects the demand. A commodity is being produced more because it constitutes the basic ingredient to the population's diet. Any increase in output of that commodity is preferable.

b. Export potential

The programs and policies delineated by the FYP actively sought to increase production of exportable products. Thus exportable crops merit a special place in the analysis.

c. Sectors whose products are being used as inputs

These sectors are the principal targets of the analysis. Special care must be given to those sectors to promote smooth growth and avoid bottlenecks.

Based on the data published in the Agricultural Statistics Yearbook, 1970, the following partition is suggested (Table 3.1).

INTERMEDIATE SECTOR

A. FOOD CROPS

1. Rice production
2. Animal food production (soybean, mungo beans, corn, ...)
3. Other food production (vegetables, potatoes, peanuts, ...)
4. Fruit production

B. INDUSTRIAL CROPS

1. Fiber crops (kenaf, jute, cotton, ...)
2. Rubber
3. Tea and coffee
4. Other industrial crops

C. LIVESTOCK PRODUCTION

1. Swine production
2. Poultry production
3. Duck raising
4. Cattle (beef cattle, buffaloes, ...)

D. FISHERIES

1. Fresh water fish
2. Salt water fish
3. Crustaceans

E. FORESTRY

1. Fuelwood
2. Lumbers
3. Cinnamon

PRIMARY SECTOR

A. LABOR

B. LAND ALLOCATION

C. IMPORTS (fertilizers, insecticides, ...)

Table 3.1. Sectors of the Agriculture

B. DATA REQUIREMENTS

Leontief pointed out that the compilation of an input-output table cannot be viewed as a strictly descriptive enterprise but must be interpreted as the fact finding component of an elaborate analytical venture.[10] The construction of the transactions table requires detailed information on the consumption of inputs and the distribution of outputs sector by sector, the analytical power of the model depends essentially on the accuracy of the estimated inter-industry flows. In compiling the table, the investigator must be aware of a certain number of problems. Conflict can arise between the conceptual definition of the object to be measured and the actual object being measured. This problem, identified as the problem of operational definitions, is inherent to all statistical problems. There also can be cases where data obtained from different sources do not agree. The problem of measurement and reconciliation is referred to the cases where the sum of inputs to the system of a particular commodity exceeds the total gross output obtained from the producing sector of that commodity. In those cases adjustments must be made and extreme care should be taken to avoid the danger of transforming the entire project into pure guesswork. The problems of statistical imputations may arise when imputation for the whole economy must be made based on observations taken in a small area of the system.

The informations entered in an input-output table consist of two different sets of numbers:

1. Outputs (X_i) and inputs (x_{ij}) to each sector
2. Technical coefficients (a_{ij})

[10] Input-Output Analysis: An Appraisal, National Bureau of Economic Research, pp. 9-28, Princeton University Press, 1955.

Inputs and Outputs

For a large part, data for outputs can be obtained directly from statistical reports issued on a yearly basis by governmental agencies. Generally, those data describe the quantity of commodity i available for consumption outside the producing industry. In cases where the production of commodity i requires a percentage of the gross output to be used as input then the reported output corresponds to the gross output minus the input x_{ii}. Then x_{ii} must be estimated and the gross output must be adjusted to this new information. An example can be found in the production of rice, a percentage of paddy produced is used as seeds for the next crop.

Data concerning inputs of primary goods can be considered as being readily available. Lands allocated to each particular crop are reported by the Agricultural Statistical Service, labor input to each sector can be obtained from the National Bureau of Labor Statistics. Fertilizer and insecticide used in each sector can be computed from the fertilizer-insecticide subsidies allocated to farmers.

Technical Coefficients

Once the aggregate inputs and aggregate output to an industry are estimated, the technical coefficients for that industry can be computed as the ratios of each individual input to the total output:

$$a_{ij} = \frac{x_{ij}}{X_j}$$

Wassily Leontief suggested an alternative procedure for the derivation of the coefficients where aggregate output and inputs cannot be easily estimated.[11] The procedure is called "derivation of coefficients by

[11] Analysis of Puerto Rican Economy, Amor Gosfield, Input-Output Analysis: An Appraisal, pp. 335-337, National Bureau of Economic Research.

engineering analysis" and consists of following the production process step by step and recording the inputs used at each stage of production. The inter-industry flows are then imported from the production function established for a random sample of firms. The production of poultry requires labor, animal foods, and some other miscellaneous expenses. Taking observations on a sample of poultry farms, the statistician can derive a production function for poultry production.

As it has been pointed out, data may disagree in certain instances; in such cases, more measurements should be made and a reconciliation on the data obtained will be necessary.

The derivation of production cost functions for the production of a wide variety of crops has been one of the projects promised in the FYP (p. 129). It can be expected that the input-output model can make use of this information. This will reduce considerably the workload of model building.

IV. STATISTICAL ESTIMATION

The values of inputs and output obtained for each industry during each time period represent observations made on the corresponding random variables. The technical coefficients computed from the ratios of inputs and outputs are one observation made on the random variables a_{ij}.

Suppose input-output tables for n successive time periods are available, then it is expected that statistical procedures can be developed to give estimates for the technical coefficients. In the following two statistical procedures will be presented. The least squares procedure assume constant technical coefficients, the time series analysis allows linear changes over time.

A. LEAST SQUARES ESTIMATION

Consider a simple model consisting of two producing sectors

$$X_1 = x_{11} + x_{12} + D_1$$

$$X_2 = x_{21} + x_{22} + D_2$$

The technical matrix is:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \left[a_{.1}, a_{.2} \right]$$

$a_{.i}$ is a column vector.

Let B be the inverse of (I-A)

$$B = (I-A)^{-1} = \frac{1}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \begin{bmatrix} (1-a_{22}) & a_{12} \\ a_{21} & (1-a_{11}) \end{bmatrix}$$

Label the matrix B as $B = \begin{vmatrix} b_{1.} \\ b_{2.} \end{vmatrix}$.

$b_{i.}$ is a row vector.

Then expressing the vector of gross output as a function of the vector of final demand gives:

$$X = (I-A)^{-1} D$$

$$\begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} b_{1.} \\ b_{2.} \end{vmatrix} X \begin{vmatrix} D_1 \\ D_2 \end{vmatrix} = \begin{vmatrix} b_{11}D_1 + b_{12}D_2 \\ b_{21}D_1 + b_{22}D_2 \end{vmatrix}$$

or explicitly

(4.1)

$$X_1 = \frac{1}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} ((1-a_{22})D_1 + a_{12}D_2)$$

$$X_2 = \frac{1}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} (a_{21}D_1 + (1-a_{11})D_2)$$

Since $a_{ij} = \frac{x_{ij}}{X_j}$ or $x_{ij} = a_{ij}X_j$

Then relations between inputs and final demands can be obtained.

(4.2)

$$\begin{cases} x_{11} = \dfrac{a_{11}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} ((1-a_{22})D_1 + a_{12}D_2) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_{22} = \dfrac{a_{22}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} (a_{21}D_1 + (1-a_{11}) D_2) \end{cases}$$

The system (4.2) can also be obtained using matrix multiplication:

Let D_t be the vector of final demands at time period t. Consider the block diagonal matrix $\underline{D}$ where:

$$\underline{D} = \begin{vmatrix} D^t & & & \\ & D^t & & \\ & & D^t & \\ & & & D^t \end{vmatrix} = \begin{vmatrix} d_1 & & & \\ d_2 & & & \\ & d_1 & & \\ & d_2 & & \\ & & d_1 & \\ & & d_2 & \\ & & & d_1 \\ & & & d_2 \end{vmatrix}$$

Let

$$\underline{B} = \begin{vmatrix} b'_{1.} & & & \\ & b'_{1.} & & \\ & & b'_{2.} & \\ & & & b'_{2.} \end{vmatrix} = \frac{1}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \begin{vmatrix} 1-a_{22} & & & \\ a_{12} & & & \\ & 1-a_{22} & & \\ & a_{12} & & \\ & & a_{21} & \\ & & 1-a_{11} & \\ & & & a_{21} \\ & & & 1-a_{11} \end{vmatrix}$$

and

$$\underline{A} = \begin{vmatrix} a_{.1} \\ \\ a_{.2} \end{vmatrix} = \begin{vmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{vmatrix}$$

Then

$$\underline{B} \times \underline{A} = \frac{1}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} \begin{vmatrix} a_{11}(1-a_{22}) \\ a_{11}a_{12} \\ a_{21}(1-a_{22}) \\ a_{21}a_{12} \\ a_{21}a_{12} \\ a_{12}(1-a_{11}) \\ a_{22}a_{21} \\ a_{22}(1-a_{11}) \end{vmatrix} = \begin{vmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{vmatrix}$$

and

$$
{}_t\underline{D} \times \underline{B} \times \underline{A} = \frac{1}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \begin{vmatrix} a_{11}(1-a_{22})D_1 + a_{11}a_{12}D_2 \\ a_{21}(1-a_{22})D_1 + a_{21}a_{12}D_2 \\ a_{12}a_{21}D_1 + a_{12}(1-a_{11})D_2 \\ a_{22}a_{21}D_1 + a_{22}(1-a_{11})D_2 \end{vmatrix} = \begin{vmatrix} {}_tx_{11} \\ {}_tx_{21} \\ {}_tx_{12} \\ {}_tx_{22} \end{vmatrix}
$$

The system (4.2) was rederived.

Note that $\underline{C} = \underline{B} \times \underline{A}$ is a matrix of constant coefficients c_i.

If t time periods worth of data are available, then t relations can be written:

$$
{}_t\underline{X} = {}_t\underline{D} \times \underline{B} \times \underline{A} = {}_t\underline{D} \times \underline{C}
$$

where

$$
{}_t\underline{X} = \begin{matrix} x_{.1} \\ \\ x_{.2} \end{matrix}
$$

The least squares procedure minimizes the expression:

$$
\text{Min} \sum_t ({}_t\underline{X} - {}_t\underline{D}\ C)^2
$$

It may be noted that the minimizing operation can be simplified considerably using the fact that the C_i's are not independent and that relations can be established between the C_i's.

In this case, the following relations can be used:

1. $\dfrac{C_1}{C_3} = \dfrac{C_2}{C_5} = \dfrac{a_{11}}{a_{21}}$

2. $C_4 = C_5$

3. $\dfrac{C_6}{C_8} = \dfrac{C_4}{C_7} = \dfrac{a_{21}}{a_{22}}$

4. $\dfrac{C_1}{C_2} \times C_6 - C_5 = 1$

Substituting those relations in the expression to be minimized reduces the number of variables to four. The algebra for the 2 x 2 case is developed in Appendix A.

In general, the coefficients C_i's to be estimated never exceeds the number of the technical coefficients.

Consider the case of n producing sectors. At the tth time period, let:

$_t x_{.i}$ be the column vector of inputs to industry i.

$a_{.i}$ be the ith column of the technology matrix.

$b_{j.}$ be the jth row the inverse matrix $(I-A)^{-1}$.

$_t\underline{D}$ be the vector of final demand.

Then construction the matrices $_t\underline{D}$, $\underline{A}$, $\underline{B}$ as:

$$_t\underline{D} = \underbrace{\begin{vmatrix} _tD' & & & & \\ & _tD' & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & _tD' \end{vmatrix}}_{m \text{ rows x } m^2 \text{ columns}} \; ; \quad \underline{A} = \underbrace{\begin{vmatrix} a_{.1} \\ a_{.2} \\ \vdots \\ a_{.n} \end{vmatrix}}_{m^2 \text{ rows x 1 column}}$$

and:

$$\underline{B} = \underbrace{\begin{vmatrix} \boxed{\begin{matrix} b'_{1.} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & b'_{1.} \end{matrix}}^{m \text{ columns}}_{m \text{ rows}} & & & & & & & & \\ & b'_{2.} & & & & & & & \\ & & \cdot & & & & & & \\ & & & \cdot & & & & & \\ & & & & b'_{2.} & & & & \\ & & & & & \ddots & & & \\ & & & & & & b'_{n.} & & \\ & & & & & & & \ddots & \\ & & & & & & & & b'_{n.} \end{vmatrix}}_{m^2 \text{ rows x } m^2 \text{ columns}}$$

As in the 2 x 2 case, relations between final demands and inputs can be obtained.

$$_t\underline{X} = {_t\underline{D}} \times \underline{B} \times \underline{A}$$

or

$$_t\underline{X} = {_t\underline{D}} \times \underline{C}$$

Minimizing $\sum_t ({_t\underline{X}} - {_t\underline{D}}\,\underline{C})^2$, the least squares estimates for $\underline{C}$ are obtained.

The estimates for the technical coefficients are computed from the relation $\overline{B} \times \overline{A} = \overline{C}$.

B. TIME SERIES ANALYSIS OF INPUT COEFFICIENTS

Assume the (i,j)th input coefficient a_{ij} is an independent random variable. In principle, the random variables a_{ij} and a_{kj} are negatively correlated. According to relation (2.6), $\sum_i a_{ij} = 1$. Then if a_{ij} increases, a_{kj} decreases for some k, $(k \neq i,\ k = 1, 2, \ldots, n)$.

Tilanus showed that for all practical purposes, these interdependencies can be neglected in the time series analysis[12].

Given the above assumption, a linear regression equation can be fitted to the time series of the technical coefficients.

Let $a_{ij}(t) = \alpha_{ij} + \beta_{ij}t + e_{ij}(t)$

$$i = 1, 2, \ldots, n$$

$$j = 1, 2, \ldots, n$$

$$t = 0, 1, 2, \ldots, T$$

where α_{ij} is a constant coefficient

β_{ij} is the trend or the slope of the regression line

$e_{ij}(t)$ is a random variable assumed to be normally distributed N(0,1)

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[12] Tilanus, C.B., Input-Output Experiments in the Netherlands, Chap. 3, pp. 36-51.

For reason of convenience, consider a translation of time so that its average is zero for the period considered.

Then again:

$$a_{ij}(t) = \alpha_{ij} + \beta_{ij} t + e_{ij}(t)$$

$$t = -\frac{T}{2}, \ldots, -1, 0, 1, \ldots, \frac{T}{2}$$

The least squares estimators of the coefficients α_{ij} and β_{ij} are

$$\alpha_{ij} = \frac{1}{T+1} \sum_{t=-T/2}^{T/2} a_{ij}(t)$$

$$\beta_{ij} = \frac{\sum_{t=-T/2}^{T/2} t \, a_{ij}(t)}{\sum_{t=-T/2}^{T/2} t^2}$$

The value of the estimate of $a_{ij}(t)$ at time t is then:

$$E\left[a_{ij}(t)\right] = E\left[\alpha_{ij} + \beta_{ij} t + e_{ij}(t)\right]$$

$$\hat{a}_{ij}(t) = \alpha_{ij} + \beta_{ij} t.$$

The linear regression estimates $\hat{a}_{ij}(t)$ satisfy the restriction (2.6); namely, they should add up columnwise to one.

$$\sum_{i=1}^{n} \hat{a}_{ij}(t) = \frac{1}{T+1} \sum_i \sum_t a_{ij}(t) + \frac{\sum_i \sum_t t \, a_{ij}(t)}{\sum_t t^2}$$

Since $\sum_i a_{ij}(t) = 1$ for fixed t and $\sum_{t=-T/2}^{T/2} t = 0$

$$\sum_i \hat{a}_{ij}(t) = \frac{1}{T+1} \sum_t (1) + \cancel{\frac{\sum_t t(1)}{\sum_t t^2}} = \frac{T+1}{T+1} = 1$$

The linear regression estimation of technical coefficients allows an extrapolation of the matrix A to a future time (t+s). It may be noted that since the a_{ij}'s and a_{kj}'s are negatively correlated, then it can

happen that one or more extrapolated values yield negative values for those coefficients. This problem can be minimized if s is small; in other words, if extrapolation is made for a very near future.

A measure of goodness of fit is provided by the t test on regression coefficients.

Let

$$(S_{ij})^2 = \frac{1}{T-1} \sum_{t=-T/2}^{T/2} \left[a_{ij}(t) - \hat{a}_{ij}(t)\right]^2$$

where $a_{ij}(t)$ is the observed value at time t.

$(S_{ij})^2$ is the estimator of the deviations of the observed values from regression values.

Under the null hypothesis H_o: there is no trend, then:

$$\frac{S^2_{ij}}{\sigma^2_{ij}} \text{ is } X^2(T-1)$$

where σ^2_{ij} is the variance of the random variable $a_{ij}(t)$. Also, β_{ij} is distributed normal mean zero and variance $\sigma^2_{ij}/ \Sigma t^2$, and

$$\frac{S_{ij}}{\sigma_{ij}/\sqrt{\Sigma t^2}} \text{ is } U(0,1)$$

Therefore, the t statistics are:

$$t_{ij} = \frac{\beta_{ij}\sqrt{\Sigma t^2}}{(S_{ij})^2} ,$$

t_{ij} is distributed Student's t with (T-1) degrees of freedom.

V. USES OF THE MODEL

The use of any inter-industry model is intended at the very first stage of the construction of the model. In the present case, the aim is to develop a forecasting device for agriculture and to provide a basis for efficient resource allocation.

The following three sections propose to look at the use of input-output models in these areas.

A. PREDICTIONS OF INTERMEDIATE DEMAND GIVEN FINAL DEMAND

Suppose there are no constraints on primary inputs; then, given a vector of final demands or bill of goods, the vector of intermediate demands can be computed.

Let

$$Z = \begin{vmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{vmatrix} = \begin{vmatrix} \sum_j x_{ij} \\ \sum_j x_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \sum_j x_{nj} \end{vmatrix}$$

be the vector of inputs to the economy.

From equation (2.6) $X = AX + D$ or $X = Z + D$. Then $Z = AX$ and using equation (2.5) $X = (I-A)^{-1} D$ gives:

$$Z = A(1 - A)^{-1} D$$

or

$$Z = \left[(A-I) + I\right] (I-A)^{-1} D$$

(5.1)

$$Z = \left[(I - A)^{-1} - I\right] D$$

Relation (5.1) gives the predictions of intermediate demands for the period t + s, using the technology matrix of time period t. Usually, this information is summarized with the use of subscript.

$$Z^{predicted}_{t+s} = \left[(I - A)^{-1}_{t} - I\right] D^{given}_{t+s}$$

A measure of the quality of predictions is given by considering the relative predictions errors:

$$\text{Relative error} = \frac{\text{Prediction - Realization}}{\text{Realization}}$$

B. <u>PREDICTIONS OF PRIMARY INPUTS DEMAND GIVEN FINAL DEMAND</u>

Let y_{kj} be the input of primary commodity k to industry j.

$$k = 1, 2, \ldots, m$$

$$j = 1, 2, \ldots, n$$

The primary input coefficient matrix is:

$$B = \left[b_{kj}\right] \quad \text{where} \quad b_{kj} = \frac{y_{kj}}{X_j}$$

B is an m x n matrix, m rows corresponding to the m primary input sectors.

Let Y be the vector of primary inputs to the system. Y is an m column vector.

$$Y = \left(\sum_{j=1}^{n} y_{kj}\right) \qquad k = 1. 2.,,,.m$$

or

$$Y = \left(\sum_{j=1}^{n} b_{kj}X_j\right)$$

or in matrix notation

(5.2) $$Y = BX$$

Given a final demand vector D, the required vector of primary inputs can be computed as follows: using (5.2) and the relation $X = (I-A)^{-1} D$ gives

$$Y = B(I-A)^{-1} D$$

The general notation is:

$$Y^{predicted}_{t+s} = B_t \, (I - A_t)^{-1} \, D^{given}_{t+s}$$

Note that not any bill of goods can be achieved. Let Y^o_{t+s} be the vector of primary inputs available for use, then Y_{t+s} is upperbounded. Y^o_{t+s} can be as:

$$Y^o_{t+s} = \begin{vmatrix} \text{total labor} \\ \text{total land} \\ \cdots\cdots \\ \text{capital} \end{vmatrix}$$

Then

$$B_t(I - A_t)^{-1} D^{given}_{t+s} \leq Y^o_{t+s}$$

Only a fraction of the given bill of goods D_{t+s} can be achieved, and it will be determined by the scarcest primary resource.

Suppose the binding constraint is land constraint, then there will be primary resources that are not being fully used. If maximum use of available resources is wanted, then a parametric analysis can be considered.

Let

$$D_{t+s} + \delta\ D_{t+s} = \begin{vmatrix} d_{1_{t+s}} + \delta_1\ d_{1t+s} \\ d_{mt+s} + \delta_m\ d_{mt+s} \end{vmatrix}$$

Then solving the equation:

$$B_t\ (I - A_t)^{-1}\ (D_{t+s} + \delta\ D_{t+s}) \leq Y^o_{t+s}$$

yields maximum use of primary resources.

The solving procedure may involve iterative schemes. A systematic procedure for solving the similar problem is given by the linear programming formulation.

C. <u>A LINEAR PROGRAMMING APPROACH TO THE PROBLEM OF RESOURCES ALLOCATIONS IN INPUT-OUTPUT ANALYSIS</u>

Let $\underline{D} = \{D^i\}$ be the set of final demand vectors that can be achieved given a primary input vector Y^o.

Consider the set $\underline{D}^* \subset D$. D^* is the set of final demand vectors such that no vector $D^i \in \underline{D}^*$ is dominated by another vector $D^k \in \underline{D}^*$. This says that if $d^i_j < d^k_j$ then $d^i_l > d^k_l$ for some l.

A criterion for choice of the final demand vector to be accepted can be defined using weights for commodities i, i = 1, 2,..., n.

Let $C = (C_1, C_2, \ldots, C_n)$ be the vector of weights attached to the final demand vector.

Then the overall weight of final demand vector D^l is $W^l = \sum_i c_i d^l_i$.

The problem becomes: Maximize the overall weight of the vector of final demand, subject to technological constraints and resources constraints.

It can be schematically written as follows[13]:

$$\text{Max } W = C\,D$$

$$\text{Subject to } (I - A)X = D$$

$$BX \le Y^o$$

$$X \ge 0$$

where

$A = (a_{ij})$ is the technology matrix

$B = (b_{ij})$ is the matrix of primary inputs coefficients

$X = \begin{vmatrix} X_1 \\ \vdots \\ X_n \end{vmatrix}$ is the vector of gross outputs

$D = \begin{vmatrix} D_1 \\ \vdots \\ D_n \end{vmatrix}$ is the vector of final demands

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[13] Gass, Saul I., Linear Programming, Chap. 11, pp. 240-247, McGraw Hill, Inc., New York, 1969. ALSO
Chenery and Clark, Inter-industry Economics, John Wiley and Sons, Inc., 1959.

$C = (C_1, \ldots, C_n)$ is the vector of weights.

and $Y^o = \begin{vmatrix} Y_1 \\ \vdots \\ Y_m \end{vmatrix}$ is the vector of primary resources.

Let $U = \begin{vmatrix} U_1 \\ \vdots \\ U_n \end{vmatrix}$ be the vector representing the unused primary resources.

Then the problem can be put into the standard form of the simplex method formulation:

$$\text{Maximize } W = C\ D$$

$$\text{Subject to } (I - A)X = D$$

$$-BX + U = -Y^o$$

The problem can also be formulated to take into account the available inventory, i.e., the quantities of final goods left over from the precedent time period.

Let $D^o = \begin{vmatrix} d_1^o \\ \vdots \\ d_m^o \end{vmatrix}$ be the vector representing the inventory.

Then the problem becomes:

$$\text{Maximize } C\ D$$

$$\text{Subject to } (I - A)X + D^o = D$$

$$- BX + U = -Y^o$$

$$X \geq 0$$

It may also be noted that the problem can be transformed to handle situations where full use of one or more primary resources is required. Suppose full employment is required, then let k_1 be the weight attached to u_1, u_1 being the variable representing the quantity of unused labor supply. Maximizing $W = CD - k_1u_1$ will solve the problem.

The formulation of the general problem is:

$$\text{Maximize } W = CD - KU$$

$$\text{Subject to } (I - A)X - D = D^o$$

$$-BX + U = -Y^o$$

$$X \geq 0$$

The explicit formulation is given in Table (5.1).

The standard simplex method formulation is given in Table (5.2)

To give an idea about the size of the problem, suppose an input-output model consists of 20 secondary sectors and three primary input sectors, then the simplex tableau will consist of:

	1	row corresponding to the objective function
	20	rows corresponding to the technical feasibility constraints $(I - A)X - D = D^o$
	3	rows corresponding to resources constraint, $-BX + U = -Y^o$
Total	24	rows

and

	20	columns corresponding to gross output variable, X_i
	20	columns corresponding to final demand variable, D_i
	3	columns corresponding to the unused primary input variable, U_i
Total	43	columns

The simplex tableau will be then of size 24 x 43. This explains why an input-output model of size less than thirty sectors have been avocated.

$$\text{Maximize } W = c_1d_1 + c_2d_2 + \ldots + c_nd_n - k_1n_1 \ldots - k_mn_m$$

Subject to:

Feasibility Constraints

$$\begin{array}{l}
(1-a_{11})\, X_1 - a_{12}X_2 \ldots - a_{1n}X_n - D_1 \ldots = D_1^o \\
-a_{21}X_1 - (1-a_{22})\, X_2 \ldots - a_{2n}X_n - D_2 \ldots = D_2^o \\
\quad\vdots \\
-a_{n1}X_1 - a_{n2}X_2 \ldots (1-a_{nn})\, X_n \ldots \quad -D_n = D_n^o
\end{array}$$

Resources Constraints

$$\begin{array}{l}
-b_{11}X_1 - b_{12}X_2 \ldots - b_{1n}X_n + U_1 = -Y_1 \\
-b_{21}X_1 - b_{22}X_2 \ldots - b_{2n}X_n \quad + U_2 = -Y_2 \\
\quad\vdots \\
-b_{m1}X_1 - b_{m2}X_2 \ldots -b_{mn}X_n \ldots \quad +U_m = -Y_m
\end{array}$$

$X_i \geq 0 \qquad i = 1, 2, \ldots, n$

$D_i \geq 0 \qquad i = 1, 2, \ldots, n$

$U_i \geq 0 \qquad i = 1, 2, \ldots, m$

Table 5.1. Formulation of the general problem

		Production Activities				Unused Resources Activities			Final Use Activities				
	Commodities	P_1	P_2	. . .	P_n	P_{n+1}	. . .	P_{n+m}	F_1	F_2	. . .	F_n	Restrictions
Technical	Commodity 1	$1 - a_{11}$	$- a_{12}$	. . .	$- a_{1n}$				-1				D_1^o
Feasibility	Commodity 2	$- a_{21}$	$1 - a_{22}$	. . .	$- a_{2n}$					-1			D_2^o
Constraints	.	.									.		.
	.	.									.		.
	.	.										.	.
	Commodity n	$- a_{n1}$	$- a_{n2}$	. . .	$1 - a_{mn}$							-1	D_n^o
Primary	Resource 1	$- b_{11}$	$- b_{12}$	. . .	$- b_{1n}$	+1							$-Y_1$
Inputs	. .	.			.		.						.
Constraints	. .	.			.		.						.
	. .	.			.		.						.
	Resource m	$- b_{m1}$	$- b_{m2}$	. . .	$- b_{mn}$			+1					$-Y_m$
	Objective Function	0	0	. . .	0	$-k_1$	. . .	$-k_m$	C_1	C_2	. . .	C_n	Maximum
	Activity Level	X_1	X_2	. . .	X_n	U_1	. . .	U_m	D_1	D_2	. . .	D_n	

Table 5.2. Simplex Formulation

VI. CONCLUSION

The purpose of this paper has been to develop a basis for the construction of an input-output model for the rural sector in Vietnam. It represents only the very first exploratory step into the field of inter-industry modeling. Likewise, the techniques proposed have been fairly simple. That should not lead into thinking that is all there is to input-output analysis. Since the introduction of the first model by Wassily Leontief thirty years ago, input-output techniques have progressed at giant steps. Input-output models now range from fairly simple time independent linear models to more complex dynamic models where time changes are explicitly taken into account.

The uses of the model have also diversified: from period wise projections and resources allocation as presented in Section V, dynamic resources allocation[14] and time optimal[15] systems have been developed.

In regard to the partitioning suggested in Table 3.1, many objections may be raised. Should the model include those industries that relates directly to the agricultural sector such as rice milling, fish sauce production, animal food processing, etc. That depends on how one defines the rural sector. Should need arise, the model can be extended to encompass those sectors. Objections may also be made as to the appropriateness of the classification. The answer is that Table 3.1 should be viewed as an example.

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[14] Eckauss, Richard S., and Parikh, K.S., Planning for Growth, pp. 85-141, The MIT Press, 1968.

[15] Andrew Brody, Optimal and Time Optimal Paths of the Economy, Contributions to Input-Output Analysis, pp. 62-74, American Elsevier Publishing Company, Inc., New York, 1970.

As it has been pointed out, a sensible partitioning can only be made with the help of some degree of expertise on the structure of the Vietnamese rural economy. Input-output is typically a group project. Experience has shown that inter-industry modeling requires a combination of diverse talents[16]. Expert knowledge and some familiarity with the economy to be modeled should be added to the skills in statistical estimation and data analysis. It is clear then that the modeling of the rural sector should be undertaken within or at least with the active participation of the Ministry of the Agriculture.

The confidence that can be placed on the values of input-output forecasts derived from the models proposed in Section IV depends directly on the validity of the assumptions on constant coefficients. If deviations from those assumptions are not too far, then accurate predictions can be expected. In the present case, although it is true that one can expect changes to occur in farming techniques, these changes will take place only gradually. Farmers in Vietnam, as anywhere else, have been known as being risk averters and will accept new technology in a very conservative way. Consequently, if predictions are being made for a near future period then the results will not be too far off.

Carl F. Christ pointed out that among other properties, input-output values are consistent; and comparing to linear regression models, input-output values are consistently more accurate[17]. Calculations made for the

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[16] Christ, Carl F, Nature and Uses of Data and Methods, Input-Output Analysis: An Appraisal, pp. 53-135, National Bureau of Economic Research, 1955. ALSO
Bharadwaj, Economic Analysis in Input-Output Framework, Input-Output Research Association, India, 1967.

[17] Christ, Carl F., Nature and Use of Data and Methods, Input-Output Analysis: An Appraisal, pp. 53-135, National Bureau of Economic Research, New York, 1955.

Netherlands economy by C.B. Tilanus showed that relative errors of input-output forecasts are consistently less than those of GNP blow ups. Moreover, the relative prediction errors for a time lag of 12 years (1948-1960) are in the order of 5 percent[18].

It should be noted that the assumptions on constant coefficients in the Netherland's model are more likely to be violated than in the present model for the rural sector. The Netherlands being a highly industrialized country should have an economy that is much more dynamic, and therefore technological changes are more frequent.

In any event, depending on the willingness to accept the assumptions on constant coefficients, a static model or a static model with updating schemes can be used. The techniques for updating the input-output model using most recent data are available[19].

In cases where drastic changes in rural economy are expected, then a dynamic model should be developed. Incidentally, in input-output analysis dynamic models are a logic continuation of the static models; they require the same kind of information, only dynamic models involve more advanced mathematics and dynamic programming solving procedures.

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[18]Tilanus, C.B., Input-Output Experiments in the Netherlands, pp. 13-18.

[19]Iden., pp. 34-123.

APPENDIX A. LEAST SQUARES ESTIMATION FOR THE 2 x 2 CASE

In Section (4.1) the relations between the C_i's have been developed as:

$$C_3 = \frac{C_1}{C_2} \times C_5$$

$$C_4 = C_5 \qquad \text{(A.1)}$$

$$C_6 = (1 + C_5)\,\frac{C_2}{C_1}$$

$$C_8 = (1 + C_5)\,\frac{C_7 C_2}{C_5 C_1}$$

The relations between C_i and a_{ij} were

$$\begin{aligned}
C_1 &= a_{11}(1-a_{22}) \\
C_2 &= a_{11}a_{12} \\
C_3 &= a_{21}(1-a_{22}) \\
C_4 &= a_{21}a_{12} \\
C_5 &= a_{12}a_{21} \\
C_6 &= a_{12}(1-a_{11}) \\
C_7 &= a_{21}a_{22} \\
C_8 &= a_{22}(1-a_{11})
\end{aligned}$$

The least squares estimation minimizes the expression:

$$\text{Min} \sum_t [X^t - D^t C]^2$$

Subject to the relations (A.1)

or

$$\text{Min} \sum_t [(x_{11}^t - D_1^t C_1 - D_2^t C_2)^2 + (x_{21}^t - D_1^t C_3 - D_2^t C_4)^2 + (x_{12}^t - D_1^t C_5 - D_2^t C_6)^2$$

$$+ (x_{22}^t - D_1^t C_7 - D_2^t C_8)^2]$$

Subject to relations (A.1)

Replacing C_3, C_4, C_6, C_8 in the objective function yields:

$$\text{Min } W = \sum_t \Big[(x_{11}^t - D_1^t C_1 - D_2^t C_2)^2 + (x_{21}^t - D_1^t \frac{C_1}{C_2} C_5 - D_2^t C_5)^2$$

$$+ (x_{12}^t - D_1^t C_5 - D_2^t (1+C_5) \frac{C_2}{C_1})^2 + (x_{22}^t - D_1^t C_7 - D_2^t (1+C_5) \frac{C_7 C_2}{C_5 C_1})^2\Big]$$

Taking the derivatives:

$$-\tfrac{1}{2} \frac{\partial W}{\partial C_1} = \sum_t (x_{11}^t - D_1^t C_1 - D_2^t C_2) D_1^t + \sum_t (x_{21}^t - D_1^t \frac{C_1 C_5}{C_2} - D_2^t C_5) D_1^t \frac{C_5}{C_2}$$

$$-\sum_t (x_{12}^t - D_1^t C_5 - D_2^t (1+C_5) \frac{C_2}{C_1^2}) D_2^t (1+C_5) \frac{C_2}{C_1^2}$$

$$-\sum_t (x_{22}^t - D_1^t C_7 - D_2^t (1+C_5) \cdot \frac{C_7 C_2}{C_7 C_5}) D_2^t (1+C_5) \frac{C_7 C_2}{C_1^2 C_5}$$

$$-\tfrac{1}{2} \frac{\partial W}{\partial C_2} = \sum_t \Big[(x_{11}^t - D_1^t C_1 - D_2^t C_2) D_2^t - (x_{21}^t - D_1^t \frac{C_1 C_5}{C_2} - D_2^t C_5) D_1^t \frac{C_1 C_5}{C_2^2}$$

$$+ (x_{12}^t - D_1^t C_5 - D_2^t (1+C_5) \frac{C_2}{C_1}) D_2^t \frac{(1+C_5)}{C_1} + (x_{22}^t - D_1^t C_7 - D_2^t (1+C_5) \frac{C_7 C_2}{C_1 C_5})$$

$$D_2^t \frac{(1+C_5)}{C_1 C_5} C_7 \Big]$$

$$-\tfrac{1}{2} \frac{\partial W}{\partial C_5} = \sum_t \Big[(x_{21}^t - D_1^t \frac{C_1}{C_2} C_5 - D_2^t C_5)(D_1^t \frac{C_1}{C_2} + D_2^t) + (x_{12}^t - D_1^t C_5 - D_2^t (1+C_5) \frac{C_2}{C_1})$$

$$(D_1^t + D_2^t \frac{C_2}{C_1}) - (x_{22}^t - D_1^t C_7 - D_2^t (1+C_5) \frac{C_7 C_2}{C_5 C_1})\Big] D_2^t \frac{C_7 C_2}{C_1} \frac{1}{C_5^2}$$

$$-\tfrac{1}{2} \frac{\partial W}{\partial C_7} = \sum_t \Big[(x_{22}^t - D_1^t C_7 - D_2^t (1+C_5) \frac{C_7 C_2}{C_5 C_1})(D_1^t + D_2^t \frac{(1+C_5) C_2}{C_5 C_1})\Big]$$

Let $\sum_t x_{11}^t D_1^t = a_1 \qquad \sum_t x_{11}^t D_2^t = b_1$

$\sum_t x_{21}^t D_1^t = a_2 \qquad \sum_t x_{21}^t D_2^t = b_2$

$$\sum_t x_{12}^t D_1^t = a_3 \qquad \sum_t x_{12}^t D_2^t = b_3$$

$$\sum_t x_{22}^t D_1^t = a_4 \qquad \sum_t x_{22}^t D_2^t = b_4$$

and $$\sum_t D_i^t D_j^t = d_i d_j$$

Then

$$-\tfrac{1}{2}\frac{\partial W}{\partial C_1} = a_1 + a_2\frac{C_5}{C_2} - \frac{1+C_5}{C_2^2}b_3 - \frac{(1+C_5)C_7C_2}{C_1^2C_5}b_4 - d_1^2C_1(1+\frac{C_5^2}{C_2^2}) + d_2^2(1+C_5)^2 \cdot$$

$$\cdot\left(\frac{1}{C_1C_2} + \frac{C_7^2C_2^2}{C_1^3C_5^2}\right) + d_1d_2\left[(1+C_5)\left(\frac{C_5}{C_2^2} + \frac{C_7^2C_2}{C_1^2C_5}\right) - C_2(1+\frac{C_5^2}{C_2^2})\right] = 0$$

$$-\tfrac{1}{2}\frac{\partial W}{QC_2} = b_1 - \frac{C_1C_5}{C_2^2}a_2 + \frac{1+C_5}{C_7}b_3 + \frac{1+C_5}{C_1C_5}b_4 + \frac{C_1^2C_5^2}{C_2^3}d_2^2$$

$$-d_1d_2[C_1(1-\frac{C_5^2}{C_2^2}) + (1+C_5)(\frac{C_5}{C_7} + \frac{C_7^2}{C_1C_5}) + d_2^2C_2\ 1+(1+C_5)(\frac{1}{C_1C_7} + \frac{C_7}{C_1^2C_5^2})] = 0$$

$$-\tfrac{1}{2}\frac{\partial W}{\partial C_5} = \frac{C_1}{C_2}a_2 + a_3 + b_2 + \frac{C_2}{C_1}b_3 - \frac{C_7C_2}{C_1C_5}b_4 - d_1^2C_5(1+\frac{C_1^2}{C_2^2})$$

$$-d_1d_2[2\frac{C_1^2+C_2^2}{C_1C_2}C_5 + \frac{C_2}{C_1}(C_5 - \frac{C_1^2}{C_5^2})] - d_2^2\left[C_5 + \frac{C_2^2}{C_1^2}(1+C_5) - C_7\frac{(1+C_5)}{C_5}\right] = 0$$

$$-\tfrac{1}{2}\frac{\partial W}{\partial C_7} = a_4C_1^2C_5^2 - d_1^2C_1^2C_5^2C_7 - 2d_1d_2(1+C_5)\,C_2C_7C_5C_1 + b_4(1+C_5)\,C_2C_5C_1$$

$$-d_2^2(1+C_5)^2\,C_7C_2^2 = 0$$

Solve for C_7

$$C_7 = \frac{[a_4C_1C_5 + b_4C_2(1+C_5)]\,C_1C_5}{d_1^2C_1^2C_5^2 + 2d_1d_2(1+C_5)C_2C_5C_1 + d_2^2(1+C_5)^2C_2^2}$$

Replacing C_7 in the three first equations, a system of 3 equations to 3 unknowns is obtained.

Suppose data are available, then an iterative search scheme can be developed. Using the starting guess values for C_1, C_2, C_5 as

$$C_1^o = a_{11}(1-a_{22})$$

$$C_2^o = a_{11}a_{12}$$

$$C_5^o = a_{12}a_{21}$$

It is hoped that the scheme will converge. Attempt has not been made because of lack of real data.

APPENDIX B. APPROXIMATION FORMULA FOR THE INVERSE OF (I - A)

If the Hawkins-Simon conditions are satisfied, then the system is viable and the inverse of (I-A) exists. $(I-A)^{-1}$ can be approximated by the power series of A.

$$(I - A)^{-1} = I + A + A^2 + A^3 + A^4 + \ldots$$

For more details, see Appendix R.5, pp. 276-293, Mathematical Economics, Kelvin Lancaster.

Consider the matrix A:

$$A = \begin{vmatrix} .2 & .2 & .2 \\ .1 & .2 & .1 \\ .2 & 0 & .1 \end{vmatrix}$$

The powers of A are:

$$A^2 = \begin{vmatrix} .10 & .08 & .08 \\ .06 & .06 & .05 \\ .06 & .04 & .05 \end{vmatrix}; \qquad A^3 = \begin{vmatrix} .044 & .036 & .036 \\ .028 & .024 & .023 \\ .026 & .020 & .021 \end{vmatrix}$$

$$A^4 = \begin{vmatrix} .0196 & .0160 & .0160 \\ .0126 & .0104 & .0103 \\ .0114 & .0092 & .0093 \end{vmatrix}; \qquad A^5 = \begin{vmatrix} .00872 & .00712 & .00712 \\ .00562 & .00460 & .00459 \\ .00506 & .00412 & .00413 \end{vmatrix}$$

$(I-A)^{-1}$ approximated to the fifth power yields

$$(I-A)^{-1} = I+A+A^2+A^3+A^4+A^5 = \begin{vmatrix} 1.37232 & .33912 & .33912 \\ .20622 & 1.29900 & .18750 \\ .30246 & .07332 & 1.18443 \end{vmatrix}$$

$$\text{The actual inverse is:} \quad \begin{vmatrix} 1.37931 & .34482 & .34482 \\ .21073 & 1.30268 & .19157 \\ .306513 & .076628 & 1.18773 \end{vmatrix}$$

APPENDIX C. A NUMERICAL EXAMPLE

Consider an input-output model consisting of three producing sectors and two primary input sectors:

Sector Producing \ Sector Consuming	Agriculture	Animal Production	Fisheries	Final Demand	Gross Output
Agriculture	16	8	6	50	80
Animal Prod.	8	8	3	21	40
Fisheries	16	0	3	11	30
				Surplus	
Labor	32	20	6	58	70
Import	8	4	12	0	24
Gross Output	80	40	30	Total	150

Hypothetical Transactions Table in Money Units

Dividing the five first rows by the last row element by element, the technology matrix A and the primary input matrix B are obtained:

$$A = \begin{bmatrix} .2 & .2 & .2 \\ .1 & .2 & .1 \\ .2 & 0 & .1 \end{bmatrix}$$

$$B = \begin{bmatrix} .4 & .5 & .2 \\ .1 & .1 & .4 \end{bmatrix}$$

$$(I-A) = \begin{vmatrix} .8 & -.2 & -.2 \\ -.1 & .8 & -.1 \\ -.2 & 0 & .9 \end{vmatrix}$$

The inverse of (I-A) is given in Appendix B.

$$(I-A)^{-1} = \begin{vmatrix} 1.37 & .34 & .34 \\ .21 & 1.30 & .19 \\ .30 & .07 & 1.19 \end{vmatrix}$$

1. Gross Output Predictions Given Final Demand

a. Unconstrained Case

Given $D = \begin{vmatrix} 60 \\ 30 \\ 20 \end{vmatrix}$ what should the vector X be?

X is given by the relation $X = (I-A)^{-1}D$.

$$X = \begin{vmatrix} 1.37 & .34 & .34 \\ .21 & 1.30 & .19 \\ .30 & .07 & 1.19 \end{vmatrix} \times \begin{vmatrix} 60 \\ 30 \\ 20 \end{vmatrix} = \begin{vmatrix} 100.2 \\ 55.4 \\ 43.9 \end{vmatrix}$$

The intermediate demand is:

$$Z = X - D = \begin{vmatrix} 100.2 \\ 55.4 \\ 43.9 \end{vmatrix} - \begin{vmatrix} 60 \\ 30 \\ 20 \end{vmatrix} = \begin{vmatrix} 40.2 \\ 25.4 \\ 23.9 \end{vmatrix}$$

b. Constrained Case

It is obvious that if the primary resources are infinite, then the proposed bill of good can be achieved. What happens if the primary input vector is constrained to a level Y^o? Suppose:

$$Y^o = \begin{vmatrix} 70 \\ 30 \end{vmatrix}$$

To achieve the given bill of good D, the primary input demand is:

$$Y = BX = \begin{vmatrix} .4 & .5 & .2 \\ .1 & .1 & .4 \end{vmatrix} \times \begin{vmatrix} 100.2 \\ 55.4 \\ 43.9 \end{vmatrix} = \begin{vmatrix} 76.56 \\ 33.12 \end{vmatrix}$$

$Y \geq Y^o$, the given bill of good is not feasible.

2. Linear Programming Formulation

Suppose a vector of weights is chosen to correspond to the final demand vector $C = (4, 3, 2)$.

Suppose that primary inputs are constrained by the vector $Y^o = \begin{matrix} 130 \\ 50 \end{matrix}$ and, to add realism, suppose that the final demand for year t+s must be at least equal to the final demand of year t.

If D_{t+s} is the final demand of year t+s then

$$D_{t+s} > D_t = \begin{vmatrix} 50 \\ 21 \\ 11 \end{vmatrix}$$

The linear programming problem can be formulated as follows:

$$\text{Maximizes } X_0 = 4\, d_1 + 3\, d_2 + 2\, d_3$$

Subject to:

technology constraints	$.8X_1 - .2X_2 - .2X_3 - d_1$	$= 0$
	$-.1X_1 + .8X_2 - .1X_3 - d_2$	$= 0$
	$-.2X_1 + .9X_3 - d_3$	$= 0$
primary inputs constraints	$.4X_1 + .5X_2 + .2X_3$	≤ 130
	$.1X_1 + .1X_2 + .4X_3$	≤ 50
demand constraints	d_1	≥ 50
	d_2	≥ 21
	d_3	≥ 11

$$X_i \geq 0;\ i = 1, 2, 3$$

The problem was solved on computer IBM 360 using Subroutine MPS/360.

<u>Solution</u> Optimal objective value $X_0 = 658.46$

Variable levels
$X_1 = 196.0$
$X_2 = 80.89$
$X_3 = 55.78$
$d_1 = 129.47$
$d_2 = 39.54$
$d_3 = 11.00$

Bounds: Labor - upper bound (used up)
Import- upper bound (used up)

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An exploratory approach to the problem of modeling the rural sector in Vietnam as an open static input-output model is attempted. A tentative classification of the agricultural economy into primary and secondary sectors is undertaken. Methods for data collection and uses are suggested. Anticipating the case where data for more than one time period are available a statistical estimation procedure for parameters is developed; a time series analysis of technical coefficients is also presented. The uses of the model as a predicting device is discussed and a linear programming approach to the

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